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QUINTESSENCE: A MINI-REVIEW

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Models where the accelerated expansion of our Universe is caused by a quintessence scalar field are reviewed. In the framework of high energy physics, the physical nature of this field is discussed and its interaction with ordinary matter is studied and explicitly calculated. It is shown that this coupling is generically too strong to be compatible with local tests of gravity. A possible way out, the chameleon effect, is also briefly investigated.

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1. Introduction

It is now established that the expansion of our Universe is accelerated [1]. However, the reason for this acceleration is still a theoretical mystery since there is no natural mechanism in the standard model of cosmology, beside the cosmological constant, which could explain this phenomenon. As is well-known, the cosmological constant required in order to explain the observations is very far from what one theoretically expects. This last fact reflects that the acceleration is characterized by an energy scale of $\sim 10^{-3}$eV which is very different from the natural scales of particle physics where new physics could show up. Many proposal have been made in order to explain this puzzle but it is fair to say that none of them has won through. In this short review article, we consider the case of quintessence, that is to say the case where a scalar field is responsible for the accelerated expansion.

This article is organized as follows. In Sec. 2, we briefly review the main features of quintessence while in Sec. 3 we discuss its origin in high energy physics. Sec. 4 is devoted to its behavior during inflation and Sec. 5 studies its coupling with ordinary matter. Finally, in Sec. 6, we present our conclusions.

2. Quintessence in brief

In this model, gravity is described by general relativity and the Universe is made of radiation, pressure-less matter and quintessence, a scalar field denoted Q in what

follows. Then, the Friedman equation reads

$$\frac{3}{a^2}\mathcal{H}^2 = \kappa\left[\frac{Q'^2}{2a^2} + V(Q) + \rho_{\rm B}\right], \tag{1}$$

where $\kappa \equiv 8\pi/m_{\rm Pl}^2$, $m_{\rm Pl}$ being the Planck mass. The quantity $a(\eta)$ is the scale factor and $\mathcal{H}$ is defined by $\mathcal{H} = a'/a$, a prime denoting a derivative with respect to conformal time. $\rho_{\rm B}$ stands for the background energy density and is either the radiation energy density or the pressure-less energy density according to the era considered. The Friedman equation must be supplemented with two conservation equations that read (here, we restrict ourselves to models of quintessence where the kinetic term is standard [2])

$$Q'' + 2\mathcal{H}Q' + a^2\frac{\mathrm{d}V(Q)}{\mathrm{d}Q} = 0\,, \quad \rho_{\rm B}' + 3\mathcal{H}\left(1+\omega_{\rm B}\right)\rho_{\rm B} = 0\,, \tag{2}$$

where $\omega_{\rm B}$ is the equation of state parameter defined to be the pressure to energy density ratio. In the case of radiation $\omega_{\rm B} = 1/3$ while $\omega_{\rm B} = 0$ for a pressure-less fluid. Already at this stage, an important assumption is made, namely that the quintessence scalar field and the background fluid are separately conserved. We will come back to this crucial question in what follows.

The only quantity which has not yet been specified is the potential $V(Q)$. Its shape should be chosen such that it gives a satisfactory model. Obviously, the first requirement to be met is to have an accelerated expansion, namely

$$\frac{\ddot{a}}{a} = -\frac{\kappa}{6}\left(\rho_{\rm cdm} + 2\rho_{\rm rad} + \rho_{\rm Q} + 3p_{\rm Q}\right) > 0\,, \tag{3}$$

where a dot means a derivative with respect to cosmic time. Clearly, this can be achieved if the quintessence field presently dominates the matter content of our Universe and if its potential $V(Q)$ is sufficiently flat so that $p_{\rm Q} < 0$. However, this is not sufficient. The acceleration should start relatively recently, *i.e.* at a redshift of order one, $z_{\rm acc} \sim \mathcal{O}(1)$, which means that, prior to $z_{\rm acc}$, Q was subdominant or, in other words, that Q was a test field. Moreover, there is also the question of the initial conditions. Which values $Q_{\rm ini}$, $Q'_{\rm ini}$ should one take and at which initial time should one impose them?

A crucial observation is that the above mentioned questions can be addressed if one assumes that the shape of $V(Q)$ is given by the so-called Ratra-Peebles potential (typically, this is in fact more general) [3]

$$V(Q) = M^{4+\alpha}Q^{-\alpha}\,, \tag{4}$$

where M is a typical energy scale and $\alpha > 0$ is a free positive index. Indeed, under the assumption that the scalar field is a test field, one can show that the Klein-Gordon (non-linear) equation possesses a particular solution which is a scaling solution given by

$$Q \propto a^{3\left(1+\omega_{\rm B}\right)/(\alpha+2)}\,, \quad \rho_{\rm Q} \propto a^{-3\left(1+\omega_{\rm Q}\right)}\,, \quad \omega_{\rm Q} = \frac{\alpha\omega_{\rm B} - 2}{\alpha+2}\,. \tag{5}$$

An interesting feature of this solution is that the equation of state parameter $\omega_{\rm Q}$ changes its value when one goes from the radiation dominated epoch to the matter dominated epoch. In some sense, the test scalar field adapts its behavior to the evolution of the scale factor. One says that it tracks the background evolution and, for this reason, we call it a tracker field. Another important property is that, according to the above equations, $\rho_{\rm Q}$ scales less rapidly than $\rho_{\rm B}$ and, hence, will eventually dominate the matter content of the Universe (in which case, clearly, the test field approximation breaks down).

A priori, despite its nice features, this solution seems of little importance because the chances to join it appears to involve a very severe fine-tuning of the initial conditions. However, this is not so because, beside being a scaling solution, it is also an attractor. In practice, if we start the evolution, say, just after inflation, this means that there is a huge range of initial conditions from which the attractor is joined before present time. This property is illustrated in Fig. 1 where the evolution of the radiation, matter and quintessence energy densities is represented. The left and the right panels corresponds to two different initial conditions for $\rho_{\rm Q}$ (chosen after reheating, at $z \sim 10^{28}$). On the left panel, the attractor is joined at $z \sim 10^5$ while, on the right one, it is joined at $z \sim 10^{10}$. Prior to the tracking regime, the evolution of $\rho_{\rm Q}$ depends on the initial conditions but, once the field is on tracks, one always converges toward the same evolution. This is why the solution (5), far from being useless, is on the contrary very important. Indeed, almost regardless of the values of $Q_{\rm ini}$, $Q'_{\rm ini}$, the evolution of $\rho_{\rm Q}$ at small redshifts (but not too small) will always be described by Eq. (5). Let us be more precise about this last statement. In fact, one can show that the allowed initial values (the initial values such that the attractor is joined before present time) are approximatively such that $10^{-37}{\rm GeV}^4 < \rho_{\rm Q} < 10^{61}{\rm GeV}^4$ where $10^{-37}{\rm GeV}^4$ corresponds to the background energy density at equality whereas $10^{61}{\rm GeV}^4$ is the background energy density at reheating. This means that the initial range encompasses about 100 orders of magnitude in energy density!

We have already signaled that the solution (5) has the nice feature to scale less quickly than the background and, hence, one is guaranteed that, eventually, quintessence will dominate and cause the acceleration of the expansion. Moreover, one can show that the redshift at which the acceleration starts is given by

$$z_{\rm acc} = \left(\frac{\Omega_{\rm Q}}{\Omega_{\rm m}}\right)^{(2+\alpha)/6} - 1\,, \tag{6}$$

and is always of order one given that $\Omega_{\rm Q} \sim 0.7$ and $\Omega_{\rm m} \sim 0.3$. Therefore, one understands why the acceleration started only recently.

Let us also mention that cosmological perturbations can be computed in this model. In particular, one can show that the quintessence field does not cluster on scales smaller than the Hubble scale because the Jeans mass is precisely given by H_0^{-1}. A complete study of the Cosmic Microwave Background anisotropy has been carried out in Ref. [7].

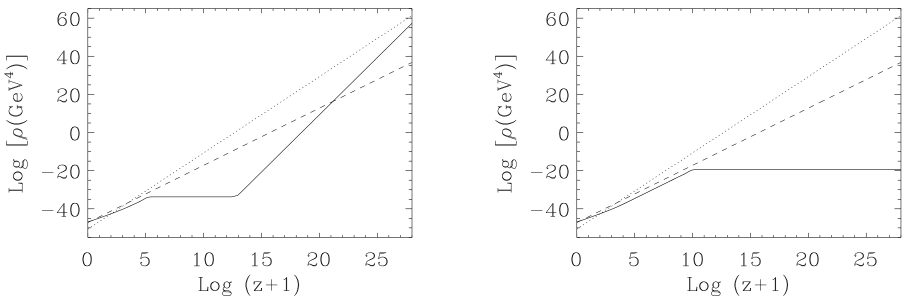

Fig. 1. Evolution of radiation (dotted line), matter (dashed line) and quintessence (solid line) energy densities for two different initial values of ρ_{Q} (left and right panels).

It should be clear that all the nice above described properties are obtained at the expense of requiring $\Omega_{\mathrm{Q}} \sim 0.7$ which, in turn, demands a (fine) tuning of the mass scale M which is easy to estimate. On the attractor the following relation holds

$$\frac{\mathrm{d}^2 V}{\mathrm{d}Q^2} = \frac{9}{2}\frac{\alpha+1}{\alpha}\left(1-\omega_{_Q}^2\right) H^2 \,. \tag{7}$$

The second derivative of the potential is nothing but the mass and can roughly be written as $\sim V/Q^2$. On the other hand, when the field is about to dominate, one has, using the Friedman equation, $H^2 \sim V/m_{\mathrm{Pl}}^2$. Therefore, given that ω_{Q} and α are of order one, the previous equation indicates that, at present time, the vacuum expectation value of the quintessence field is approximatively

$$\langle Q\rangle_{\mathrm{today}} \sim m_{\mathrm{Pl}} \,. \tag{8}$$

As a consequence, since the quintessence energy density is today of the order of the critical energy density, one can write that $V \sim M^{4+\alpha} m_{\mathrm{Pl}}^{-\alpha} \sim \rho_{\mathrm{cri}}$ and this leads to

$$\log_{10}[M(\mathrm{GeV})] \simeq \frac{19\alpha - 47}{4\alpha+4} \,. \tag{9}$$

One can check numerically that the above equation is a good approximation. The corresponding plot is represented in Fig. 2. One notices that, for, say, $\alpha > 6$, M is above the TeV scale. What has been achieved is reminiscent of the see-saw mechanism: thanks to the inverse power-law shape of the Ratra-Peebles potential, one can justify the appearance of a very small scale (by particle physics standards) even if the characteristic scale of the problem is large. Therefore, it seems that we have gained something. Of course, the question is now to justify the inverse power-law shape of the potential which was necessary in order to obtain the nice properties described before. We turn to this question in the next section.

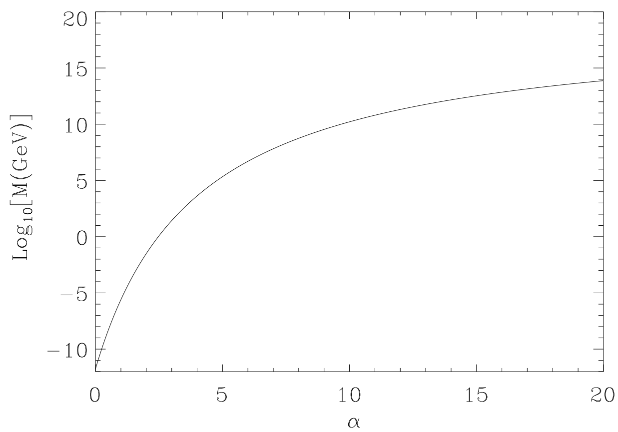

Fig. 2. The quintessence energy scale M as a function of the index α according to Eq. (9).

3. Model Building

We have explained in the previous section how the potential (4) allows us to construct an interesting model describing the accelerating Universe. It is clearly not sufficient to postulate the existence of the quintessence field and to assume a shape for its potential. One would like to understand the nature of this field in high energy physics. Obviously, there is no candidate for the quintessence field in the standard model of particle physics and, therefore, one has to seek beyond, in the extensions of the standard model. The most natural extensions are the super-symmetric ones and one will focus on this class of models in the following.

At this stage, the fact that $\langle Q\rangle_{\rm today} \sim m_{\rm Pl}$ plays an important role [4,5,6]. It means that one should consider supergravity (SUGRA) models as opposed to global super-symmetric theories [4,5]. Supergravity is a theory where super-symmetry is gauged (made local) and its predictions usually differ from those of global super-symmetry, the difference being of the order of the vacuum expectation values of the fields in the problem measured in Planck unit. Here, since $\langle Q\rangle_{\rm today}/m_{\rm Pl} \sim 1$, these corrections are of order one and it is mandatory to take them into account. This conclusion is very important and has far-reaching consequences that will be discussed in the rest of this article, the main issue at stake being whether the assumptions exposed previously and necessary in order to build a sensible quintessence model can be preserved in a SUGRA framework. We will see that the answer to this question is negative.

One can illustrate the previous claim on the following example. Can we recover the potential (4) in a SUGRA framework? We postulate that the Kähler potential and the super-potential in the quintessence sector can be Taylor expanded and are

given by [4,5]

$$K(X,Y,Q) = XX^\dagger + QQ^\dagger + \kappa^p YY^\dagger \left(QQ^\dagger\right)^p , \quad W(X,Y,Q) = gX^2Y , \quad (10)$$

Here X and Y are two charged fields under an (anomalous) $U(1)$ symmetry with charges 1 and -2, while Q is the neutral quintessence field. The constant g is a dimensionless coupling constant and p is a free coefficient. Then, one assumes that

$$\langle X\rangle = \xi , \quad \langle Y\rangle = 0 . \quad (11)$$

As a specific example, ξ can be realized as a Fayet-Iloupoulous term arising from the Green–Schwarz anomaly cancellation mechanism. In supergravity, negative contributions to the scalar potential arise from the vacuum expectation value of the super-potential. In the present situation, $\langle W_{\rm quint}\rangle = 0$ and we are guaranteed that the potential is positive definite. We are now in a position where the scalar potential can be computed. In supergravity, it is given by $V = {\rm e}^G \left(G^A G_A - 3\right)/\kappa^2$ and, in the present situation, reduces to

$$V(Q) = {\rm e}^{\kappa Q^2/2+\kappa\xi^2} \frac{M^{4+2p}}{Q^{2p}} , \quad (12)$$

where the mass scale M characterizing the potential can be expressed as $M^{4+2p} \equiv 2^p g^2 \xi^4 \kappa^{-p}$. Firstly, we see that we do not recover the potential (4) but that the SUGRA corrections play an important role: they have been exponentiated and appear in the prefactor. Phenomenologically, it turns out to be an advantage since the equation of state $\omega \equiv p_{\rm Q}/\rho_{\rm Q}$ can be closer to -1 than with the Ratra–Peebles potential. Secondly, this illustrates the fine-tuning of the parameters in a new way. Indeed, we showed previously that the mass M can be large by particle physics standard which is certainly a nice feature. However, we see here that there is still a somehow hidden fine-tuning problem. Indeed, if one uses the expression of the mass obtained before and assume that the coupling constant g is of order one (in order to avoid a fine-tuning), then one has $\xi \sim \rho_{\rm cri}^{1/4}$, *i.e.* a tiny scale. Therefore, despite the satisfactory value of M, there is still a severe fine-tuning of $\langle X\rangle$.

4. The Quintessence field during inflation

We have seen that one of the main advantage of the quintessence scenario is that it is insensitive to the choice of the initial conditions. More precisely, if the energy density after inflation, at the beginning of the subsequent radiation-dominated era, is such that $10^{-37}{\rm GeV}^4 < \rho_{\rm Q} < 10^{61}{\rm GeV}^4$, then the attractor is joined. It is a huge range but, nevertheless, it is finite. Since this range refers to the value of the quintessence energy density after inflation, one can wonder how Q evolves during inflation and whether this evolution can drive $\rho_{\rm Q}$ away from the allowed range.

As we are going to see, many effects can influence the behavior of Q during inflation. For the purpose of illustration, let us assume that the inflaton potential is given by $V_{\rm inf} = V_0 \left(\phi/m_{\rm Pl}\right)^n$ (large field model) and that Q and ϕ do not interact.

Almost by definition, Q is a test field during inflation and its evolution can be found by solving the Klein-Gordon equation. One arrives at

$$\frac{Q}{m_{\rm Pl}}=\left\{\left(\frac{Q_{\rm ini}}{m_{\rm Pl}}\right)^{\alpha+2}+\frac{\alpha(\alpha+2)}{n(2-n)}\frac{M^{4+\alpha}m_{\rm Pl}^{-\alpha}}{V_0}\left[\left(\frac{\phi_{\rm ini}}{m_{\rm Pl}}\right)^{2-n}-\left(\frac{\phi}{m_{\rm Pl}}\right)^{2-n}\right]\right\}^{1/(\alpha+2)}, \tag{13}$$

for $n\neq 2$ while, for $n=2$, the result reads

$$\frac{Q}{m_{\rm Pl}}=\left[\left(\frac{Q_{\rm ini}}{m_{\rm Pl}}\right)^{\alpha+2}+\frac{\alpha(\alpha+2)}{2}\frac{M^{4+\alpha}m_{\rm Pl}^{-\alpha}}{V_0}\ln\frac{\phi_{\rm ini}}{\phi}\right]^{1/(\alpha+2)}. \tag{14}$$

As a matter of fact, for any value of n we obtain that $Q\simeq Q_{\rm ini}$ at all times, that is to say the field is frozen, if the initial value satisfies the constraint

$$\left(\frac{Q_{\rm ini}}{m_{\rm Pl}}\right)^{\alpha+2}\gtrsim\left(\frac{M^{4+\alpha}m_{\rm Pl}^{-\alpha}}{V_0}\right)\left(\frac{\phi_{\rm ini}}{m_{\rm Pl}}\right)^{2-n}. \tag{15}$$

In this case the initial conditions at the beginning of inflation directly corresponds to those after reheating.

Unfortunately, things are not that simple. There are at least two other effects that are a priori present. The first effect is that the quantum behavior of the quintessence field during inflation must be taken into account [8,9]. Indeed the various quantum kicks undergone by the quintessence field during inflation could be such that, at the end of inflation, Q has drifted away too much and is outside the allowed range or, more precisely, has a low probability of falling in this allowed range. This can be checked by means of the stochastic inflation formalism. In this approach the quintessence field becomes a stochastic quantity the evolution of which is controlled by the Langevin equation

$$\frac{{\rm d}Q}{{\rm d}t}+\frac{V'(Q)}{3H(\phi)}=\frac{H^{3/2}(\phi)}{2\pi}\xi_{\rm Q}(t)\,, \tag{16}$$

where $\xi_{\rm Q}$ is the quintessence white-noise field such that $\langle\xi_{\rm Q}(t)\xi_{\rm Q}(t')\rangle=\delta(t-t')$. Of course, the quantum effects for the inflaton field must also be taken into account. This means that ϕ is also a stochastic quantity described by its own Langevin equation. However, one can show that its influence is not important [9].

In Ref. [9], the Langevin equation for the quintessence field has been solved for the Ratra-Peebles potential and the mean value, variance and probability distribution function of Q have been computed. Then, a given model will be accepted if a large part of its probability distribution calculated at the end of inflation is contained within the allowed range, that is to say

$$\frac{Q_{\rm min}}{m_{\rm Pl}}\equiv 10^{-107/\alpha}<\frac{Q}{m_{\rm Pl}}<\frac{Q_{\rm max}}{m_{\rm Pl}}\equiv 10^{-9/\alpha}\,. \tag{17}$$

The corresponding probability has been computed in Ref. [9] and is presented in Fig. 3 for $n=2$ and $\alpha=6$. The main result is that there is an upper bound

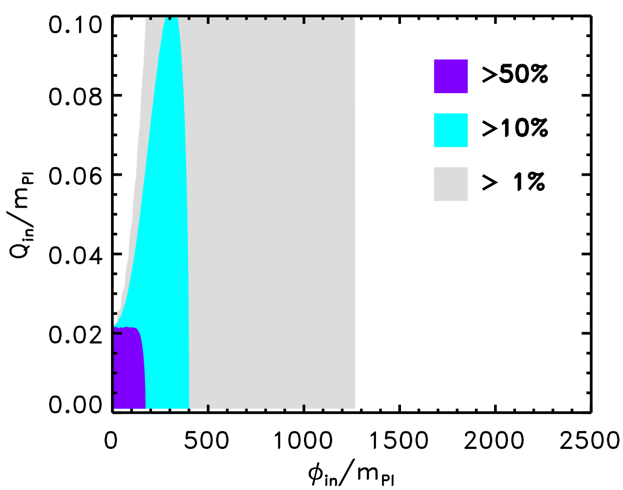

Fig. 3. Probability contours in the initial conditions plane $(Q_{\rm ini}, \phi_{\rm ini})$ of having the quintessence vacuum expectation value in the allowed range at the end of inflation. The plot has been obtained for $n = 2$ and $\alpha = 6$.

of the initial value of the inflaton field if one wants the quintessence field to be on tracks today. This is equivalent to a constraint on the total number of e-folds during inflation, very roughly speaking

$$N_{\rm T} \lesssim 10^{20(\alpha-2)/[\alpha(n+2)]}\,. \tag{18}$$

This can be easily understood. If inflation lasts too long, then the quantum quintessence field undergoes the quantum kicks during a very long period and, therefore, the probability of falling within the allowed range is small. The conclusion is that having Q on tracks today put relatively stringent constraints on the initial conditions at the beginning of inflation. Hence, when inflation is taken into account, the insensitivity to the initial conditions of the quintessence scenario does not seem to be as efficient as usually claimed.

There is still another effect which must be taken into account. So far, we have assumed that the quintessence field and the inflaton are not coupled. However, in SUGRA, as we shall see, this is not a reasonable assumption and Q and ϕ must necessarily interact [10]. Since we have argued that quintessence must be described in SUGRA, for consistency, inflation must also be described in this framework. In addition, we consider here large field models and the fact that inflation occurs for $\phi > m_{\rm Pl}$ reinforces the previous argument. In the inflaton sector, we consider a class of models described by the following Kähler potential

$$K_{\rm inf} = -\frac{3}{\kappa}\ln\left[\kappa^{1/2}\left(\rho+\rho^\dagger\right) - \kappa\mathcal{K}\left(\phi-\phi^\dagger\right)\right] + \mathcal{G}\left(\phi-\phi^\dagger\right)\,, \tag{19}$$

where $\mathcal{K}$ and $\mathcal{G}$ are given by

$$\mathcal{K} = -\frac{1}{2}\left(\phi - \phi^\dagger\right)^2 , \quad \mathcal{G} = +\frac{1}{2}\left(\phi - \phi^\dagger\right)^2 , \tag{20}$$

and where ρ represents, for instance, a modulus of a string compactification. The super-potential $W_{\rm inf} = W_{\rm inf}\left(\rho, \phi\right)$ is taken to be

$$W_{\rm inf}(\rho,\phi) = \frac{\alpha}{2} m\phi^2 . \tag{21}$$

Then, straightforward calculations lead to

$$V_{\rm inf}(\rho,\phi) = \frac{1}{\Delta^2(3-\Delta)}\alpha^2 m^2\phi^2 , \tag{22}$$

where $\Delta = \kappa^{1/2}(\rho + \rho^\dagger)$. It is easy to see that the modulus can be stabilized if $\Delta = 2$. In this case, the potential takes the form

$$V_{\rm inf}(\phi) = \frac{\alpha^2}{4} m^2\phi^2 , \tag{23}$$

which is nothing but the usual chaotic inflation potential if one chooses $\alpha = \sqrt{2}$.

Then, the most simple assumption is that the quintessence and inflation sectors are decoupled, *i.e.* that the total Kähler potential and super-potential can be written as

$$K = K_{\rm quint}\left(X,Y,Q\right) + K_{\rm inf}\left(\rho,\phi\right) , \quad W = W_{\rm quint}\left(X,Y,Q\right) + W_{\rm inf}\left(\rho,\phi\right) , \tag{24}$$

where the quintessential Kähler potential and super-potential have been given in the previous section. The next step consists in computing the scalar potential. Applying the standard SUGRA formalism one obtains $V\left(\phi, Q\right) = V_{\rm inf} + V_{\rm quint} + V_{\rm inter}$ with [10]

$$V_{\rm inter} \propto \frac{m^2}{m_{_{\rm Pl}}^4}\phi^4 Q^2 . \tag{25}$$

As announced, we have a coupling between the inflaton and the quintessence field. This simple calculation is interesting because it exemplifies many interesting properties: even if the two fields live in different sectors, they interact. Technically, this is because, in the formula $V = {\rm e}^{G}\left(G^A G_A - 3\right)/\kappa^2$, the G multiplies $G^A G_A$. Physically, this is because, in SUGRA, the various fields always interact through gravity even if they live in different sectors. This is why the coupling constant is the Planck mass and why the coupling is said to be Planck suppressed (see the $m_{_{\rm Pl}}^{-4}$ in the above equation). However, because $\langle\phi\rangle \sim m_{_{\rm Pl}}$, the coupling is not small.

What is the effect of the coupling on the evolution of Q during inflation? Does it drive Q away from the allowed range? From the quintessence point of view, there is a time dependent effective potential given by $V_{\rm quint} + V_{\rm inter}$. One can show that, after a transitory regime, Q quickly settles at the bottom of this potential and, then, just follows the time-dependent minimum of the effective potential

$$Q_{\rm min}(N) = m_{_{\rm Pl}}\left\{\frac{\alpha}{32\pi^2}\left(\frac{H_0}{m_{_{\rm Pl}}}\right)^2\left(\frac{m}{m_{_{\rm Pl}}}\right)^{-2}\left[\frac{\phi(N)}{m_{_{\rm Pl}}}\right]^{-4}\right\}^{1/(\alpha+2)} . \tag{26}$$

This is true regardless of the initial conditions at the beginning of inflation. The above solution is thus an attractor and this sets the initial conditions for the quintessence field after inflation. For $\alpha = 6$, which is our fiducial model, one has at the end of inflation: $Q_{\rm min}(N = N_{\rm T}) \simeq 1.9 \times 10^{-14} m_{\rm Pl}$. The effect of the non-renormalizable interaction is therefore to force the quintessence field to remain small during inflation. This time, the conclusion is positive since, typically, these values are within the allowed range.

5. Interacting Quintessence

We have just seen that, because SUGRA is "universal", the inflaton field and the quintessence field necessarily interact. Clearly, this conclusion calls into question our basic starting assumption, namely that quintessence today does not interact with the rest of the world, see Sec. 2 where the quintessence stress-energy tensor was separately conserved. We expect the same causes to produce the same effects: the quintessence field must interact with the ordinary matter fields [11,12]. The question is now to prove it (rather than just assuming it in a toy model) and to compute explicitly the form of this interaction for a given model. The most generic approach is to consider that there are three different sectors in the theory: the observable (where ordinary – electrons, quarks, Higgs etc ... – matter and cold dark matter live), hidden (where super-symmetry is broken) and quintessence sectors. As a consequence, the hypothesis of separate sectors implies that the Kähler and super potentials are given by the following expressions

$$K = K_{\rm quint} + K_{\rm hid} + K_{\rm obs}\,, \quad W = W_{\rm quint} + W_{\rm hid} + W_{\rm obs}\,. \tag{27}$$

Again, despite that quintessence and ordinary matter live in separate sectors, one expects them to interact. In addition, one expects their interaction to be controlled by $m_{\rm Pl}$ and to be Planck suppressed. However, since $\langle Q\rangle \sim m_{\rm Pl}$ today, this does not mean that the strength of this interaction will be small. This interaction can be computed explicitly if we are given a model, for instance, if one conservatively assumes the observable sector to be described by the mSUGRA model [13]. All the corresponding consequences have been studied in detail in Refs. [11,12]. Here, we just recap some of the main conclusions.

An important effect is that the electroweak symmetry breaking is affected by the presence of the quintessence field. In the mSUGRA model, there are two $\mathrm{SU}(2)_{\rm L}$ Higgs doublets

$$H_{\rm u} = \begin{pmatrix} H_{\rm u}^{+} \\ H_{\rm u}^{0} \end{pmatrix}\,, \quad H_{\rm d} = \begin{pmatrix} H_{\rm d}^{0} \\ H_{\rm d}^{-} \end{pmatrix}\,, \tag{28}$$

that have opposite hyper-charges, $Y_{\rm u} = 1$ and $Y_{\rm d} = -1$ with a super-potential given by $W_{\rm obs} = \mu H_{\rm u} \cdot H_{\rm d}$. Usually, the fermions acquire a mass through their interaction with the Higgs bosons. The mass is proportional to the Yukawa coupling and to Higgs vacuum expectations values. In presence of quintessence, because of

the unavoidable interaction of Q with $H_{\rm u}$ and $H_{\rm d}$, the vacuum expectation values of $H_{\rm u}$ and $H_{\rm d}$, $v_{\rm u}$ and $v_{\rm d}$, become Q-dependent (that is to say time-dependent), namely

$$v_{\rm u}(Q) = \frac{v(Q)\tan\beta(Q)}{\sqrt{1+\tan^2\beta(Q)}} \quad v_{\rm d}(Q) = \frac{v(Q)}{\sqrt{1+\tan^2\beta(Q)}}\,, \tag{29}$$

where $v \equiv \sqrt{v_{\rm u}^2 + v_{\rm d}^2} \sim 174\mathrm{GeV}$ and $\tan\beta \equiv v_{\rm u}/v_{\rm d}$. It implies that fermion masses become Q dependent. Moreover there are two kinds of masses, depending on whether the fermions couple to $H_{\rm u}$ or $H_{\rm d}$

$$m_{\rm u}^{\rm F}(Q) = \lambda_{\rm u}^{\rm F} {\rm e}^{\kappa K_{\rm quint}/2 + \sum_i |a_i|^2/2} v_{\rm u}(Q)\,, \tag{30}$$

$$m_{\rm d}^{\rm F}(Q) = \lambda_{\rm d}^{\rm F} {\rm e}^{\kappa K_{\rm quint}/2 + \sum_i |a_i|^2/2} v_{\rm d}(Q)\,, \tag{31}$$

where $\lambda_{{\rm u},a}^{\rm F}$ and $\lambda_{{\rm d},a}^{\rm F}$ are the Yukawa coupling of the particle coupling either to $H_{\rm u}$ or $H_{\rm d}$. The coefficients a_i describe the hidden sector, see Ref. [11,12] for more details. The time-dependence of the fermions mass has drastic consequences that we briefly review in the following.

It was claimed before that, once a model is given, the form of the interaction between quintessence and the rest of the world can be determined. Let us illustrate this on the example where the quintessence sector is described by the model presented in Sec. 2. Then, one can demonstrate [12] that $\tan\beta(Q)$ can be expressed as

$$\tan\beta(Q) \simeq \frac{\delta_1 + \delta_2\kappa Q^2 + \delta_3\kappa^2 Q^4}{\delta_4 + \delta_5\kappa Q^2}\left[1 + \sqrt{1 + \frac{(\delta_4 + \delta_5\kappa Q^2)^2}{(\delta_1 + \delta_2\kappa Q^2 + \delta_3\kappa^2 Q^4)^2}}\right]\,, \tag{32}$$

where the coefficients δ_1, δ_2, δ_3, δ_4 and δ_5 can easily be evaluated in terms of the physical parameters characterizing the model from the previous equations, μ, $m_{3/2}^0$ (gravitino mass) and $m_{1/2}^0$ (scalar mass) given at the GUT scale. The expression of the scale $v(Q)$ can also be obtained from the minimization of the Higgs potential along the lines described in Ref. [11]. One obtains

$$v(Q) = \frac{2}{\sqrt{g^2 + g'^2}} {\rm e}^{\kappa K_{\rm quint}/2} \sqrt{\left| |\mu|^2 + m_{H_{\rm u}}^2 \right|} + \mathcal{O}\left(\frac{1}{\tan\beta}\right)\,, \tag{33}$$

with the following expression for $m_{H_{\rm u}}$, coming from the renormalization group equations, see Ref. [14]

$$\begin{aligned} m_{H_{\rm u}}^2(Q) = m_{H_{\rm d}}^2(Q) - 0.36\left(1 + \frac{1}{\tan^2\beta}\right)\Biggl\{ & \left(m_{3/2}^0\right)^2\left(1 - \frac{1}{2\pi}\right) + 8\left(m_{1/2}^0\right)^2 \\ & + \left(0.28 - \frac{0.72}{\tan^2\beta}\right)\left[A(Q) + 2m_{1/2}^0\right]^2 \Biggr\}\,, \end{aligned} \tag{34}$$

$$m_{H_{\rm d}}^2(Q) = \left(m_{3/2}^0\right)^2\left(1 - \frac{0.15}{4\pi}\right) + \frac{1}{2}\left(m_{1/2}^0\right)^2\,, \tag{35}$$

and

$$A(Q) = M_{\rm S}\left(1+\frac{\kappa Q^2}{3}\right)\,, \quad B(Q) = M_{\rm S}\left(1+\frac{\kappa Q^2}{2}\right)\,, \tag{36}$$

$M_{\rm S}$ being the super-symmetric breaking scale. Notice that A and B follow the same universal relationship as in the mSUGRA model despite the presence of the quintessence field. As promised, everything is now explicit. One may notice that the coupling appears to be much more complicated than the simple forms usually assumed in the literature.

Let us now investigate the consequences of having time-dependent fermion masses. Firstly, there will be a fifth force. If the mass of the quintessence field is less than 10^{-3}eV, the range of the force is such that it can be experimentally seen. In order not to to be in contradiction with fifth force experiments such as the recent Cassini spacecraft experiment, its strength must be small and one must require the parameter $\alpha_{\rm u,d}$ defined by

$$\alpha_{\rm u,d}(Q) \equiv \left\vert\frac{1}{\kappa^{1/2}}\frac{{\rm d}\ln m_{\rm u,d}^{\rm F}(Q)}{{\rm d}Q}\right\vert \tag{37}$$

to be such that $\alpha_{\rm u,d}^2 \le 10^{-5}$ [15]. This result is valid for a gedanken experiment involving the gravitational effects on elementary particles. For macroscopic bodies, the effects are more subtle. Generically, in the models presented above, this limit is violated and quintessence is in trouble.

Secondly, we have violations of the weak equivalence principle. This is due to the fact that, in the mSUGRA model, the fermions couple differently to the two Higgs doublets $H_{\rm u}$ and $H_{\rm d}$. Violations of the weak equivalence principle are quantified in terms of the $\eta_{\rm AB}$ parameter defined by [15]

$$\eta_{\rm AB} \equiv \left(\frac{\Delta a}{a}\right)_{\rm AB} = 2\frac{a_{\rm A}-a_{\rm B}}{a_{\rm A}+a_{\rm B}}\,, \tag{38}$$

for two test bodies A and B in the gravitational background of a third one E. Current limits [15] indicate that $\eta_{\rm AB} = (+0.1 \pm 2.7 \pm 1.7)\times 10^{-13}$. Again, this limit applies only for $m_{\rm Q} < 10^{-3}$eV.

Thirdly, another consequence of the interaction between dark energy and the observable sector is the variation of the gauge couplings, depending on the complexity of the underlying model, more details on this specific question can be found in Ref. [11].

Fourthly, another consequence of having Q-dependent masses is that, *a priori*, the energy density of cold dark and baryonic matters no longer scales as $1/a^3$ but as

$$\rho \sim \frac{1}{a^3}\sum_a n_a m_{\rm u,d}^{\rm F}\left(\frac{Q}{m_{\rm Pl}}\right)\,, \tag{39}$$

where n_a is the number of non-relativistic particles. It has been shown that this type of interaction between dark matter and dark energy can result in an effective dark energy equation of state less than -1 [16,17].

Another consequence is the so-called chameleon effect [18,19,20,21]. The equation (39) implies that the effective potential for the quintessence field is modified by matter and becomes $V_{\rm eff}(Q) = V_{\rm DE}(Q) + f(Q)\rho_{\rm mat}$. This potential is explicitly time-dependent and usually possesses a minimum if $V(Q)$ has a runaway shape and $f(Q)$ is increasing with Q. At the minimum, the time-dependent mass is given by $m_{\rm Q}^2 = (V_{\rm eff})_{,QQ}$. This implies two new effects. Firstly, the mass of the field will depend on the environment (hence the name chameleon) through $\rho_{\rm mat}$. If $\rho_{\rm mat}$ is large then the mass can be such that $m_{\rm Q} > 10^{-3}$eV, thus evading all the constraints on the fifth force or on the weak equivalence principle. Secondly, there is the so-called thin shell effect. If we consider a situation where the gravitational experiments are performed on a spherical body of radius $R_{\rm b}$ embedded in a surrounding medium, then the above effective potential is not the same inside the body and outside because $\rho_{\rm matter}$ is different. As a matter of fact, this modifies the gravity tests. Indeed, one can calculate the profile of the quintessence field inside and outside the body and show that the acceleration felt by a test particle is given by

$$a = -\frac{Gm_{\rm b}}{r^2}\left[1+\frac{\alpha_{\rm Q}\left(Q_\infty - Q_{\rm b}\right)}{\Phi_{\rm N}}\right], \tag{40}$$

where $\Phi_{\rm N} = Gm_{\rm b}/R_{\rm b}$ is the Newtonian potential at the surface of the body. $Q_{\rm b}$ and Q_∞ denote the value of the field inside and outside the body respectively and the quantity $\alpha_{\rm Q}$ has been defined in Eq. (37). Therefore, we see that the parameter which controls the deviation from the Newtonian acceleration is

$$\frac{\alpha_{\rm Q}\left(Q_\infty - Q_{\rm b}\right)}{\Phi_{\rm N}}, \tag{41}$$

while, in absence of the thin shell mechanism, this would be $\alpha_{\rm Q}$ [notice that the limit $Q_\infty - Q_{\rm b} \to 0$ cannot be taken in Eq. (41) because the presence of the thin shell was assumed from the very beginning]. Hence, even if $\alpha_{\rm Q}$ is quite large, which would *a priori* rule out the corresponding model, if the new factor $\left(Q_\infty - Q_{\rm b}\right)/\Phi_{\rm N}$ is small then the model can be compatible with local tests of gravity.

6. Conclusions

In this article, we have presented a short review on quintessence. The main conclusion is that, from the high energy physics point of view, it is difficult to build, at the same time, a model which is interesting from the cosmological point of view (different from a pure cosmological constant) and compatible with the local gravity tests.

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